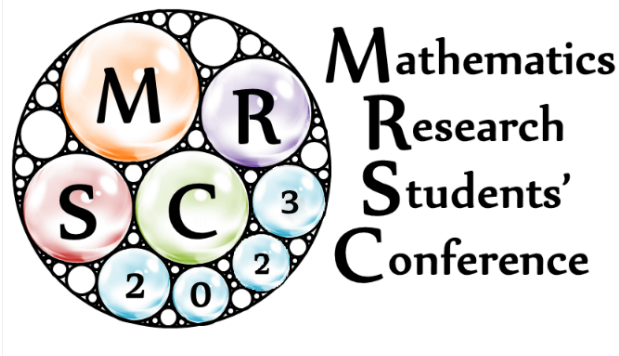
A Mathematical Interlude

Calum Hughes

The University of Manchester

29th September 2023



Research at the University of Manchester

The University of Manchester has a great research output...

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YOU NEED 27 TICKETS TO GUARANTEE A WIN ON THE UK NATIONAL LOTTERY

DAVID CUSHING AND DAVID I. STEWART

ABSTRACT. In the UK National Lottery, players purchase tickets comprising their choices of six different numbers between 1 and 59. During the draw, six balls are randomly selected without replacement from a set numbered from 1 to 59. A prize is awarded to any player who matches at least two of the six drawn numbers. We identify 27 tickets that guarantee a prize, regardless of which of the 45,057,474 possible draws occurs. Moreover, we determine that 27 is the optimal number of tickets required, as achieving the same guarantee with 26 tickets is not possible.

How the lottery works

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- 6 balls are chosen in succession.
- (Plus a bonus ball, but we'll forget about that for the purposes of today)

A Basic Calculation

of distinct tickets $=$

A Basic Calculation

of distinct tickets $= 59$

A Basic Calculation

$$\#\text{ of distinct tickets} = 59 \times 58$$

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$$\text{\# of distinct tickets} = 59 \times 58 \times 57 \times 56 \times 55 \times 54$$

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Therefore:

Lemma

If you buy $45,057,474$ *lottery tickets, you are guaranteed to win the jackpot.*

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Lemma

If you buy $45,057,474$ *lottery tickets, you are guaranteed to win the jackpot.*

However, your profit will be

$$£3,800,000 - (45,057,474 \times £2) = -£86,314,948.$$

Prizes

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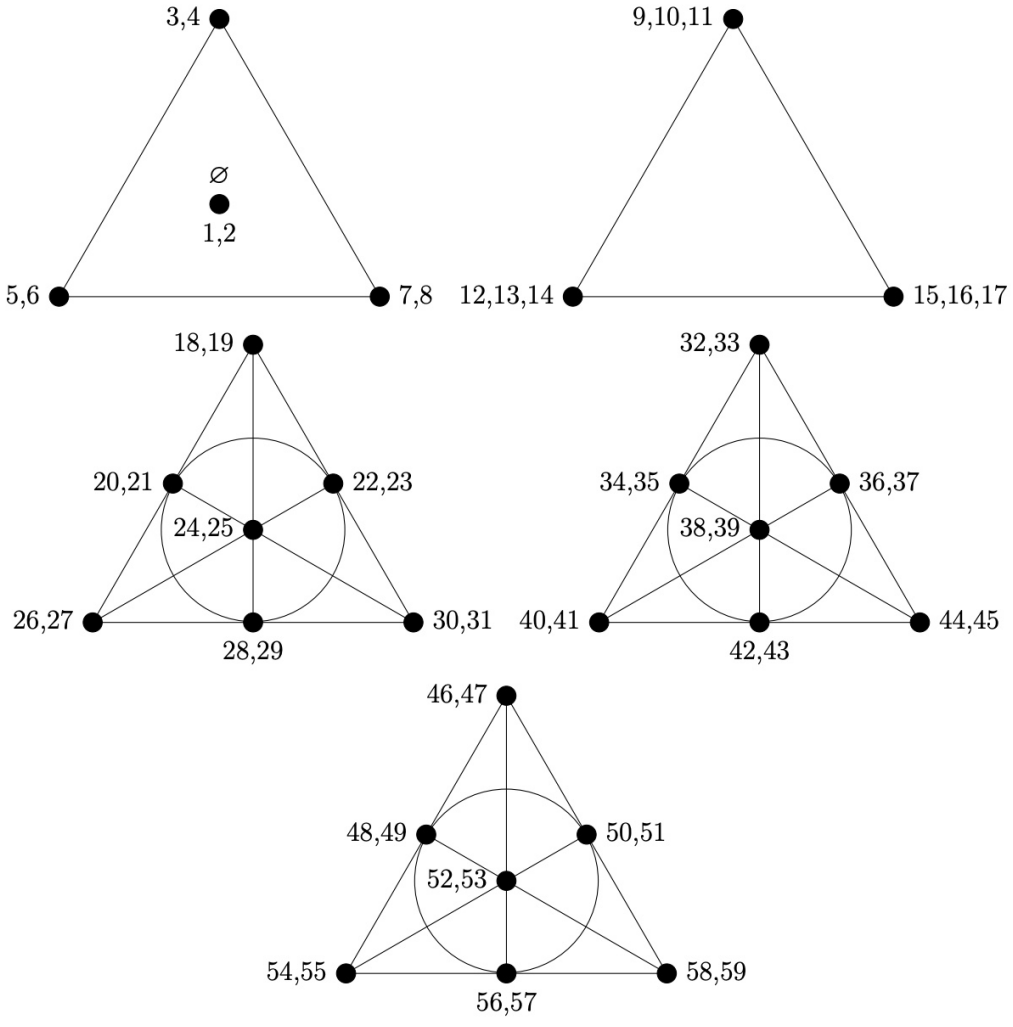
How many tickets guarantee that we get 2 matching numbers?
(We know that it is less than or equal to 45, 057, 474)

Towards Graph Theory and Combinatorics

The approach used in [CS23] reformulates this question hyper-graph theoretically in order to apply techniques from graph theory and combinatorics to tackle this problem.

Some wild Fano planes!



A figure from [CS23].

The lottery tickets

1, 2, 3, 4, 5, 6	9, 10, 11, 12, 13, 14	18, 19, 20, 21, 26, 27	32, 33, 34, 35, 40, 41	46, 47, 48, 49, 54, 55
1, 2, 3, 4, 7, 8	9, 10, 11, 15, 16, 17	18, 19, 22, 23, 30, 31	32, 33, 36, 37, 44, 45	46, 47, 50, 51, 58, 59
1, 2, 5, 6, 7, 8	12, 13, 14, 15, 16, 17	18, 19, 24, 25, 28, 29	32, 33, 38, 39, 42, 43	46, 47, 52, 53, 56, 57
		20, 21, 22, 23, 28, 29	34, 35, 36, 37, 42, 43	48, 49, 50, 51, 56, 57
		20, 21, 24, 25, 30, 31	34, 35, 38, 39, 44, 45	48, 49, 52, 53, 58, 59
		22, 23, 24, 25, 26, 27	36, 37, 38, 39, 40, 41	50, 51, 52, 53, 54, 55
		26, 27, 28, 29, 30, 31	40, 41, 42, 43, 44, 45	54, 55, 56, 57, 58, 59

A table from [CS23].

The main Theorem

Theorem (Cushing, Stewart 2023 [CS23])

The lowest number of tickets that guarantees a win on the UK lottery is 27.

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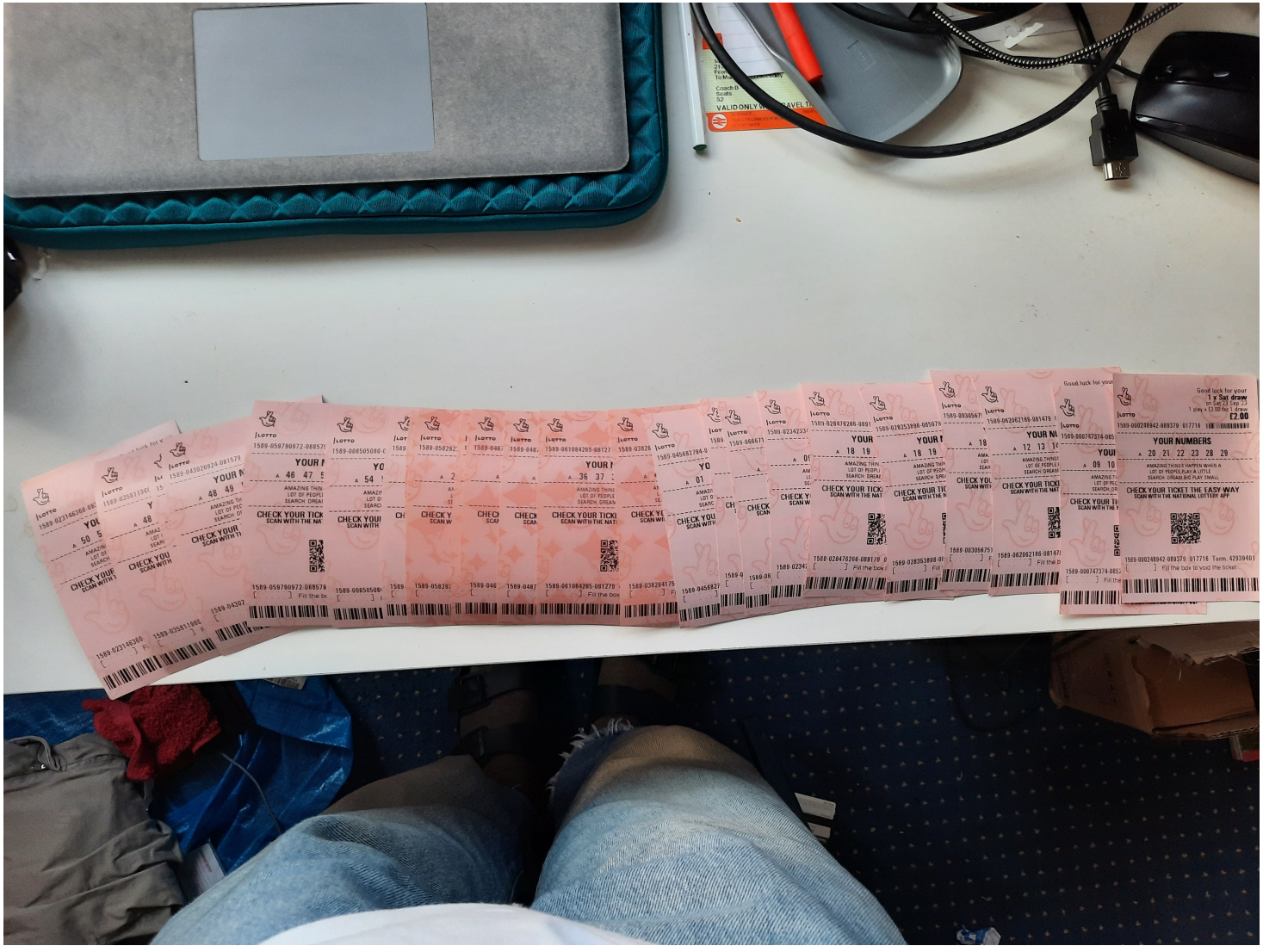
Corollary

(At least) one of you has won the lottery.

TICKETS


LOTTO
CHECK YOUR TICKET
SCAN WITH THE NATIONAL LOTTERY APP
YOUR NUMBERS

An Outing

Figure: me being laughed at in different shops

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Past Attempts

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- On June 30th 2023, these exact numbers would have profited £1810.
- On 1st July 2023, the authors of [CS23] tried this experiment. They won 3 lucky dips, all of which came to nothing. Therefore, there was a loss of £54.
- For 16th September 2023, these numbers would have profited £6 (two 3-matches = £60) and had 1 lucky dip.

The Lottery, 23rd September 2023

34

The Lottery, 23rd September 2023

34 58

The Lottery, 23rd September 2023

34 58 12

The Lottery, 23rd September 2023

34 58 12 37

The Lottery, 23rd September 2023

34 58 12 37 41

The Lottery, 23rd September 2023

34 58 12 37 41 55

If we win BIG...

WE’RE RICH
YEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEEE

If we win not so big...

The house always wins. Don’t gamble.

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- 5% of the time you will make a profit. Most of this time it will be small.

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- 75% of the time, you will only get 2 numbers to match.
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- 5% of the time you will make a profit. Most of this time it will be small.

It’s probably not worth it...


David Cushing and David I Stewart.
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arXiv preprint arXiv:2307.12430, 2023.